This exam contains 8 pages (including this cover page) and 9 questions. The total number of points is 82.

This is a unit test on Sequences and Series. More specifically, this covers arithmetic sequences and series, geometric sequences and series, as well as infinite series. All questions should be possible to complete with only the knowledge gained from videos for Precalculus 11 on www.TrevTutor.com. Hopefully, this will be a little bit harder than what your in-class test will be like. Test solutions are posted as a separate file. There is a formula sheet on the back page.

If you’d like to see more practice exams, check out www.TrevTutor.com

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**Grades**

A > 70.5
70.5 > B > 60
60 > C+ > 55
55 > C > 49.5
49.5 > C- > 41
41 > F

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<tr>
<th>Question</th>
<th>Points</th>
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<tr>
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Note: 1 question was very challenging, so 20% was possible.
This is bonus.
1. (9 points) Given the arithmetic sequence 

\[ 37, 30, 23, 16, \ldots \]

(a) (3 points) Find \( t_{12} \)

\[
t_{12} = 37 + (-7)(11) \\
= 37 - 77 \\
= -40
\]

(b) (3 points) Determine \( S_8 \)

\[
S_8 = \frac{8}{2}(2(37) + (-7)(7)) \\
= 4(74 - 49) \\
= 4(25) \\
= 100
\]

(c) (3 points) Find the general term \( t_n \)

\[
ts_n = a + d(n-1) \\
= 37 + (-7)(n-1) \\
= 37 - 7n + 7 \\
= 44 - 7n
\]
2. (9 points) Given the geometric sequence

\[ a = 30, \quad r = \frac{2}{3} \]

\[ 30, 20, 13.33, \ldots \]

(a) (3 points) Find \( t_7 \)

\[ t_7 = 30 \left( \frac{2}{3} \right)^6 = 2.634 \]

(b) (3 points) Determine \( S_7 \)

\[ S_7 = 30 \left( \frac{\left( \frac{2}{3} \right)^7 - 1}{\frac{2}{3} - 1} \right) = \frac{30 \left( -0.9415 \right)}{-\frac{1}{3}} = 84.733 \]

(c) (3 points) Determine \( S_{\infty} \)

\[ S_{\infty} = \frac{30}{1 - \frac{2}{3}} = \frac{30}{\frac{1}{3}} = 90 \]

If \( |r| < 1 \)}
3. (10 points) If an arithmetic sequence has \( t_3 = 25 \), and \( t_6 = 46 \), find the general term \( t_n \).

\[
\begin{align*}
t_4 &= 25 + d(3) \\
46 &= 25 + 3d \\
21 &= 3d \\
\frac{7}{3} &= d
\end{align*}
\]

\[
\begin{align*}
t_n &= a + d(n-1) \\
t_4 &= 11 + 7(n-1) \\
&= 11 + 7n - 7 \\
&= 4 + 7n
\end{align*}
\]

4. (5 points) If the geometric sequence has \( t_1 = 4 \) and \( t_{13} = 16,384 \), find the common ratio \( r \).

\[
\begin{align*}
t_n &= ar^{n-1} \\
t_{13} &= 4r^{13-1} \\
16,384 &= 4r^{12} \\
4096 &= r^{12} \\
\pm 2 &= r
\end{align*}
\]

(2) if only wrote \( \pm 2 \) as the common ratio.
5. (9 points) Are the following series convergent or divergent?
   (a) (3 points) $30 + 27 + 24.3 + \ldots$
   \[ r = \left| \frac{9}{10} \right| < 1 \]
   **Convergent**

   (b) (3 points) $1 - 1.5 + 2.25 - \ldots$
   \[ r = \left| \frac{-3}{2} \right| > 1 \]
   **Divergent**

   (c) (3 points) $1 + \frac{9}{10} + \frac{9^2}{10^2} + \ldots$
   \[ r = \left| \frac{9}{10} \right| < 1 \]
   **Convergent**

6. (10 points) In an arithmetic series, $S_{12} = 186$ and $t_{20} = 83$. What is $S_{40}$?

   \[
   t_n = a + (n-1)d \\
   S_n = \frac{n}{2} [2a + (n-1)d] \\
   t_{20} = a + d(20-1) \\
   83 = a + 19d \\
   S_{12} = \frac{12}{2} [2a + d(12-1)] \\
   186 = 6(2a + 11d) \\
   3s = a + 11d \\
   3 \cdot 83 = 2a + 11d \\
   27d = 135 \\
   d = 5 \\
   a = 83 - 95 \\
   a = -12 \\
   S_{40} = \frac{40}{2} [2(-12) + 5(39)] \\
   = 20(-24 + 195) \\
   = 3420 \\
   \]

   **Note:** This question is very involved. If you got it right, you understand this chapter very well. Give yourself 20/10 if you got it right; 15/10 - partially right; 10/10 - wrote anything down.
7. (10 points) A concert hall has 10 seats in the first row and 12 seats in the second row. If each row has 2 more seats than the previous row, and there are 30 rows of seats, how many seats are in the entire concert hall?

Arithmetic Sequence: 10, 12, ...

\[ t_{30} = 10 + 2(29) \]
\[ = 10 + 58 \]
\[ = 68 \]
\[ \text{# of seats in row 30} \]

\[ S_{30} = \frac{30}{2}(10 + 68) \]
\[ = 15(78) = 1170 \text{ seats total} \]

8. (10 points) A culture initially has 2,000 bacteria, and the number increases by 5% every hour.

(a) (5 points) How many bacteria are there after 8 hours?

not quite the same formula

\[ t_8 = 2000(1.05)^8 \]
\[ = 2954.92 \]
round down for # of living things
\[ = 2954 \]

(b) (5 points) Determine a formula for the number of bacteria present after \( n \) hours.

\[ t_n = 2000(1.05)^n \]
\[ \uparrow \text{start} \uparrow \text{increase per hour} \]
\[ \text{# of hours} \]
9. (10 points) A ball is dropped from 6m. After each bounce, the ball loses 25% of its height. What is the total vertical distance traveled after the ball completes its 4th bounce?

\[
\text{Rise} = \frac{4.5(10.75)^4 - 1}{0.75 - 1} = 12.305 \text{m}
\]

\[
\text{Fall} = \text{Same as rise + initial fall} = 12.305 + 6 = 18.305 \text{m}
\]

Total
\[
= 12.305 + 18.305 = 30.61 \text{m}
\]
Formulas - It’s up to you to know what they are, and how to use them!

Sequences and Series

Arith \[
\begin{align*}
t_n &= a + d(n - 1) \\
S_n &= \frac{n}{2}(a + t_n)
\end{align*}
\]
also \[S_n = \frac{n}{2} \left(2a + d(n-1)\right)\]

Geom \[
\begin{align*}
t_n &= ar^{n-1} \\
S_n &= \frac{a(r^n - 1)}{r-1} \\
S_\infty &= \frac{a}{1-r} \text{ if } |r| < 1
\end{align*}
\]